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Tugas 4 Metode Matematika (D)

2). Buktikan : $\Delta \sin(a+bx) = 2\sin\frac{b}{2}\cos\left(a+\frac{b}{2}+bx\right)$

$$\Delta \sin(a+bx) = \sin(a+b(x+1)) - \sin(a+bx)$$

※ Ingat ! $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$$\Delta\sin(a+bx) = 2.\cos\left[\frac{a+bx+b+a\ \ bx}{2}\right]\sin\left[\frac{a+bx+b-a-bx}{2}\right]$$

$$= 2\cos\left[\frac{\cancel{2}(a+bx+\frac{1}{2}b)}{\cancel{2}}\right]\sin\left[\frac{b}{2}\right]$$

$$= 2\cos\left(a+bx+\tfrac{1}{2}b\right)\sin\left(\tfrac{b}{2}\right)$$

$$\Delta\sin(a+bx) = 2\sin\left(\frac{b}{2}\right)\cos\left(a+\frac{b}{2}+bx\right)$$ [PROVED]

$\Rightarrow$ Asal persamaan $\sin A - \sin B = 2\cos\left[\frac{A+B}{2}\right]\sin\left[\frac{A-B}{2}\right]$

$$\begin{aligned} \sin(A+B) &= \sin A\cos B + \cos A\sin B \\ \sin(A-B) &= \sin A\cos B - \cos A\sin B \\ \hline \sin(A+B)-\sin(A-B) &= 2\cos A\sin B \end{aligned} \quad -$$

misal : $A+B = P$ .... (1) $\longrightarrow A = P-B$ ..... (3)

$A-B = Q$ .... (2) $\longrightarrow B = A-Q$ .... (4)

Dari (2) dan (3)

$A-B = Q$

$P-B-B = Q$

$-2B = Q-P$

$B = \frac{P-Q}{2}$

Dari (1) dan (4)

$A+B = P$

$A+A-Q = P$

$2A-Q = P$

$2A = P+Q$

$A = \frac{P+Q}{2}$

Diperoleh : $\sin(P) - \sin(Q) = 2\cos\left[\frac{P+Q}{2}\right]\sin\left[\frac{P-Q}{2}\right]$

6). Buktikan $\Delta^n x^{(n)} = n!$

Gunakan Induksi Matematika

- Buktikan $P(1)$ benar

$\Rightarrow \Delta^1 x^{(1)} = 1!$

$\Delta x = x + 1 - x$

$= 1$

Karena $1 = 1!$, maka $P(1)$ benar

- Anggap $P(k)$ benar

$\Delta^k x^{(k)} = k!$

- Buktikan $P(k+1)$ benar

$\Delta^{k+1} x^{(k+1)} = (k+1)!$

$\Delta (\Delta^k x^{(k)}) = (k+1)!$

$\Delta k! = (k+1)!$ [Terbukti]

1).

Ubah fungsi polinomial ke Polinomial Faktorial

11). $f(x) = x^3 - x + 1$

Tuliskan Koefisien dari yang terbesar :

	1	0	-1	1	$x^{(0)}$
1		1	1		
2	1	1	0	$x^{(1)}$	
		2			
	1	3	$x^{(2)}$		
	1	$x^{(3)}$			

Jadi, dalam bentuk polinomial faktorial :

$$f(x) = x^{(3)} + 3x^{(2)} + 0x^{(1)} + x^{(0)}$$

※ Pembuktian !

$\Rightarrow x(x-1)(x-2) + 3x(x-1) + 0x + 1$

$= (x^2 - x)(x-2) + 3x^2 - 3x + 1$

$= x^3 - 3x^2 + 2x + 3x^2 - 3x + 1$

$= x^3 - x + 1$ [PROVED]

13). $f(x) = 3x^3 - 7x^2 + 8x - 1$

	3	-7	8	-1	$x^{(0)}$
1		3	-4		
	3	-4	4	$x^{(1)}$	
2		6			
	3	2	$x^{(2)}$		
	3	$x^{(3)}$			

Jadi, dalam bentuk polinomial faktorial:

$$f(x) = 3x^{(3)} + 2x^{(2)} + 4x^{(1)} - x^{(0)}$$

※ Pembuktian !

$\Rightarrow 3(x)(x-1)(x-2) + 2(x)(x-1) + 4x - 1$

$= 3(x^2 - x)(x-2) + 2x^2 - 2x + 4x - 1$

$= 3x^3 - 9x^2 + 6x + 2x^2 + 2x - 1$

$= 3x^3 - 7x^2 + 8x - 1$

15). $f(x) = x^4 - 2x^3 - x$

	1	-2	0	-1	0	$x^{(0)}$
1		1	-1	-1		
	1	-1	-1	-2		$x^{(1)}$
2		2	2			
	1	1	1			$x^{(2)}$
3		3				
	1	4				$x^{(3)}$
	1					$x^{(4)}$

Jadi, dalam bentuk polinomial faktorial :

$$f(x) = x^{(4)} + 4x^{(3)} + x^{(2)} - 2x^{(1)} + 0\,x^{(0)}$$

Pembuktian !

$\Rightarrow x(x-1)(x-2)(x-3) + 4x(x-1)(x-2) + x(x-1) - 2x + 0$

$= (x^2-x)(x^2-5x+6) + 4(x^2-x)(x-2) + x^2 - x - 2x$

$= x^4 - 5x^3 + 6x^2 - x^3 + 5x^2 - 6x + 4x^3 - 8x^2 - 4x^2 + 8x + x^2 - x - 2x$

$= x^4 - 2x^3 + 0x^2 - x$

$= x^4 - 2x^3 - x$ [PROVED]